Journal of Natural Sciences Research
ISSN 2224-3186 (Paper) ISSN 2225-0921 (Online)
Vol.8, No.6, 2018
www.iiste.org

Numerical Simulation of Turbulent Natural Convection of Air in a Rectangular Enclosure

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Abstract
Turbulent natural convection of air inside a rectangular enclosure is studied numerically. The vertical surfaces of the enclosure are kept at constant temperature difference of 40K, with the hot surface at 313K and the opposite cold surface at 273K. The other surfaces are adiabatic. The finite volume based solver Fluent 6.3.26 with Boussinesq approximation was used to conduct the numerical study. The $k - \omega$ two equation RANS – based turbulence model was used for turbulent simulation. The velocity contours, streamlines, and isotherms have been studied on varying the aspect ratios of 0.5, 1 and 2 of the enclosure. Results have been represented graphically and from the interpretation, we can see that increasing the aspect ratio increases eddies at the top of the hot surface and at the bottom of the cold surface and the flow on the two vertical surfaces
Keywords: Natural Convection, Turbulence, Aspect Ratio, Numerical simulation

1. Introduction

The study of heat transfer by convection is concerned with the calculation of rates of heat transfer exchange between fluids and solids boundaries. External mechanisms can cause fluid motion e.g. by use of a pump a process called forced convection. However if the motion of the fluid is as a result of density differences which are caused by temperature difference existing in the fluid mass, the process is called free convection or Natural convection. An increase in temperature causes the fluid to expand and thus the density (mass per unit volume) decreases. Warm fluids therefore become more buoyant than cooler ones. Buoyant forces cause denser parts of the fluid to move downwards and less dense parts to move upwards. The warm fluid, initiating convective currents, then replaces the cool fluid.

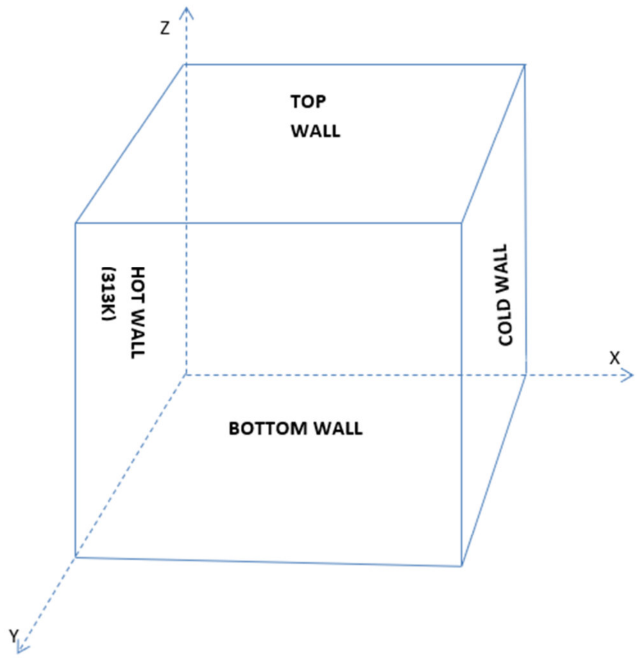
A number of researchers have based their study on this area. Brito *et al* (2003) studied turbulent natural convective heat transfer in a square enclosure, for a two – dimensional, incompressible, and unsteady flow. Cheng *et al* (2006) numerically investigated fluid flow and heat transfer characteristics of mixed convection in three – dimensional rectangular channel with four heat sources. Aminossadati and Ghasemi (2009) numerically investigated mixed convection heat transfer in a two – dimensional horizontal channel with an open cavity.

Ciafrini *et al* (2012) did a numerical study of laminar natural convection heat transfer inside rectangular enclosures partially heated from below and cooled at the top at one side, filled with either a gas or a liquid. In this study, numerical simulation was done on Rayleigh number on the cavity width ranging from 10^2 to 10^7, a Prantl number of 0.7 – 700 range, and the fraction of the heated part of the bottom cavity (E; 0.2 – 0.8. Otieno, A. K. (2012) assessed the performance of three numerical turbulence models; **k – ε, k – ω and k – ω – SST** in predicting heat transfer due to natural convection in an air filled cavity. A numerical study of natural convection of heat transfer in a three dimensional square cavity was done by Okwoyo *et al* (2013) with an aim of examining the velocity flow and temperature flow. Billah *et al* (2014) investigated mixed convection heat transfer characteristics within a ventilated square cavity having a heat-generating circular solid cylinder, with the cylinder was placed at different positions in the cavity. Siddiki (2015) numerically analyzed the steady natural convection phenomena of air in a square cavity with bottom locations of the heating portion, evaluations done at four different heating locations on the bottom wall and the local heat source on the bottom wall. Shemeret and Miroshnichenko (2015) used the finite volume method to investigate three – dimensional transient natural convection in a cubic enclosure having finite thickness solid walls subject to opposing and horizontal temperature gradient.

2. Mathematical Formulation

In this study, natural convection turbulence is studied numerically by solving the system in the figure below. The problem in study is a two – dimensional flow of an air in a rectangular enclosure if height H and length L. The enclosure is heated on the hot wall and the other side is cooled. All the boundaries of the enclosure are stationary, non – slip and impermeable. The other walls are adiabatic. The hot and cold walls of the cavity are isothermal at (313 K) and (273K) respectively

Z



This temperature $\nabla T = T_H - T_C$ brings about the movement of the fluid inside the enclosure. Where T_H is the temperature of the hot wall and T_c the cold wall temperatures with $T_H > T_C$. This implies that the density gradient of the fluid is normal to the gravity and thus the buoyancy driven natural convection will start immediately the heat is supplied.

3. Governing Equations

3.1The Equation of continuity

The equation of continuity is derived based on the principle of conservation of mass that states that mass in an isolated system is neither created nor destroyed. In other words, the amount of mass in a control volume remains constant

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = 0 \tag{1}$$

For a steady flow, equation (1) becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

3.2 Momentum equation

Using the Newton's second law of motion, which states that the net force acting on the control volume is equal to the mas times the acceleration of the fluid element within the control volume, the momentum equation can be obtained as

$$\frac{\partial}{\partial t}\rho u_i + \frac{\partial}{\partial x_j}(\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \rho g_i + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] \tag{3}$$

3.3 Energy equation

The Energyfrom the first law of thermodynamics, which states that the rate of energy increase in a closed system is equal to the work done on the system and the heat added to the system. Assuming no external heat sources, the energy equation is written as

$$\frac{\partial}{\partial t}\rho C_p T + \frac{\partial}{\partial x_j}(\rho C_p u_j T) = \frac{\partial}{\partial x_j}\left(\lambda \frac{\partial T}{\partial x_j}\right) + \beta T\left(\frac{\partial p}{\partial t} + \frac{\partial u_j p}{\partial x_j}\right) + \phi \tag{4}$$

Where ϕ denotes the dissipation function

4. Nature of Turbulence

4.1 Reynolds Decomposition

This is a mathematical technique used to separate the average and fluctuating parts of a quantity, separating the instantaneous value of a variable , for example for a variable ,ϕ, Reynolds decomposition separates it into the mean value ($\bar{\phi}$) and the fluctuating value (ϕ').

$$\phi(x_i, t) = \bar{\phi}\,(x_i) + \phi'(x_i, t) \tag{5}$$

Where the time averaged value is given by

$$\overline{\Phi} = \frac{1}{\Delta t}\int_{t_0}^{t_0+\Delta t} \phi(x_i, t)dt \tag{6}$$

Applying equation (6), equation (5) becomes,

$$\bar{\phi}(x_i, t) = \bar{\phi}(x_0) \tag{7}$$

The final set of equations (1), (3) and (4) is written as

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho U_j + \overline{\rho u_j}\right) = 0 \tag{8}$$

$$\frac{\partial}{\partial t}\left(\rho U_i + \overline{\rho u_i}\right) + \frac{\partial}{\partial x_i}\left(\rho U_i U_j + U_i \overline{\rho u_j}\right) = -\frac{\partial P}{\partial x_i} + \rho g_i + \frac{\partial}{\partial x_j}\left(\tau_{ij} - U_i\overline{\rho u_j} - \rho\overline{u_i u_j} - \overline{\rho u_i u_j}\right) \tag{9}$$

$$\frac{\partial}{\partial t}\left(C_p \rho T + C_p \overline{\rho T}\right) + \frac{\partial}{\partial x_j}\left(c_p \rho U_j T\right) = \frac{\partial p}{\partial t} + U_j \frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_i}\left[\lambda \frac{\partial T}{\partial x_j} - C_P \overline{u_i t} - C_P \overline{\rho u_i t}\right] + \phi \tag{10}$$

5. Non – Dimensionalization

In the above governing equations, there are more constants than the number of equations. Non – dimensionalization reduces the number of model parameters that govern the flow of incompressible fluids. The following set of general set of scaling variables have been used in dimensionalization

$X_J = X_J' L_R$	$U_J = U_J' U_*$	$P = P' P_R$
$\Theta = \frac{T - T_*}{\Delta T_*}$	$K = K' U_*$	$\varepsilon = \varepsilon' \frac{U_*^3}{L_R}$
$\mu = \mu' \mu_R$	$t = t' \frac{L_R}{U_*}$	$\mu_s = \mu_s' \mu_R$
$V = V' \mu_R$	$\rho = \rho' \rho_R$	$C_P = C_R C_{PR}$
$\lambda = \lambda' \lambda_R$		

Equations 8, 9 and 10 are written in non – dimensional form as

$$\frac{\partial \rho}{\partial t} + \frac{\partial\left(\rho U_j + \overline{\rho u_j}\right)}{\partial x_j} = 0 \tag{11}$$

$$\frac{\partial}{\partial t}\left(\rho U_i + \overline{\rho u_i}\right) + \frac{\partial}{\partial x_j}\left(\rho U_i U_j + U_i \overline{\rho u_j}\right) = -\left(\frac{P_R}{\rho_R U_*^2}\right)\frac{\partial P}{\partial x_i} + \left(\frac{g L_R}{U_*^2}\right)\rho g_i + \frac{\partial}{\partial x_j}\left[\left(\frac{\mu_R}{\rho_R U_* L_R}\right)\tau_{ij} - U_j\overline{\rho u_j} - \rho u_i u_j - \overline{\rho u_i u_j}\right] \tag{12}$$

$$\frac{\partial}{\partial t}\left(C_P \rho \theta + C_P \overline{\rho \Theta}\right) + \frac{\partial}{\partial x_j}\left(C_P \rho u_j \Theta\right) = \left(\frac{P_R}{C_{PR}\rho_R \Delta T_*}\right)\left[\frac{\partial P}{\partial t} + u_i \frac{\partial P}{\partial x_j} + \overline{u_j \frac{\partial P}{\partial x_j}}\right] + \frac{\partial}{\partial x_j}\left(\left\{\frac{\lambda_R}{C_{PR}\rho_R U_* L_R}\right\}\lambda \frac{\partial \Theta}{\partial x_j} - C_P \overline{\rho u_i \Theta} - C_P \overline{\rho u_i \Theta}\right) + \left(\frac{\mu_R U_*}{C_{PR}\rho_R \Delta T_* L_R}\right) + \phi \tag{13}$$

6. Methods of Solution

The finite volume based solver Fluent 6.3.26 with Boussinesq approximation is used to solve the above governing equations.

6.1 Temperature and Boundary Conditions

$$\Theta = constant, \quad \frac{\partial \Theta}{\partial n} = 0, \quad \Theta_{hot} = 1 \; and \; \Theta_{cold} = 0, v = u = 0, \frac{\partial \psi}{\partial n} = 0$$

I also used the following constants

Density of air = 1.1275 kg/m^3, Dynamic Viscosity = 1.9148e^{-5}, Kinematic Viscosity = 1.6982e^{-5}, C_p= 1.0069e^3, conductivity K= 0.027076, P_r = 0.71207, Thermal Diffusivity = 2.3848e^{-5}, Thermal Expansion Coefficient = 3.2934e^{-3}

7. Results and Discussion

7.1 Velocity Contours

Results in this study shows velocity contours in terms of velocity magnitude and stream function in different aspect ratio. The velocity magnitude contours are concentrated on the right top side of the hot wall of the enclosure and the bottom right of the cold wall. An increase in the aspect ratio increases the number of eddies in the top side of the hot wall and also at the bottom side of the cold surface with the highest velocity contour at a velocity of 0.845 m/s.

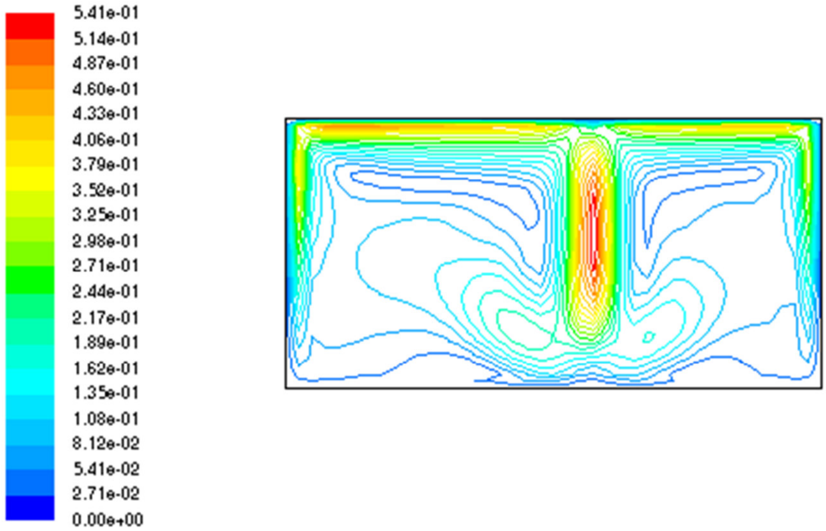


Fig 7.1 Ar = 0.5 velocity magnitude

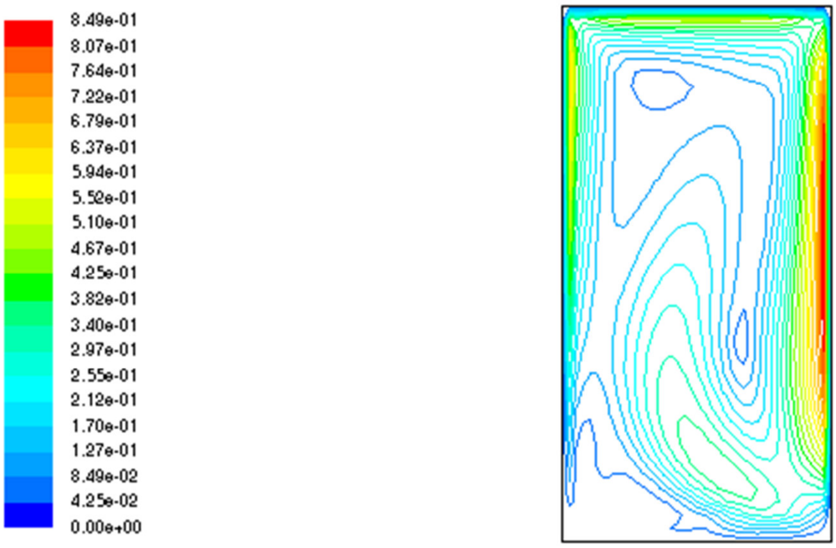


Fig 7.2 Ar = 1 velocity magnitude

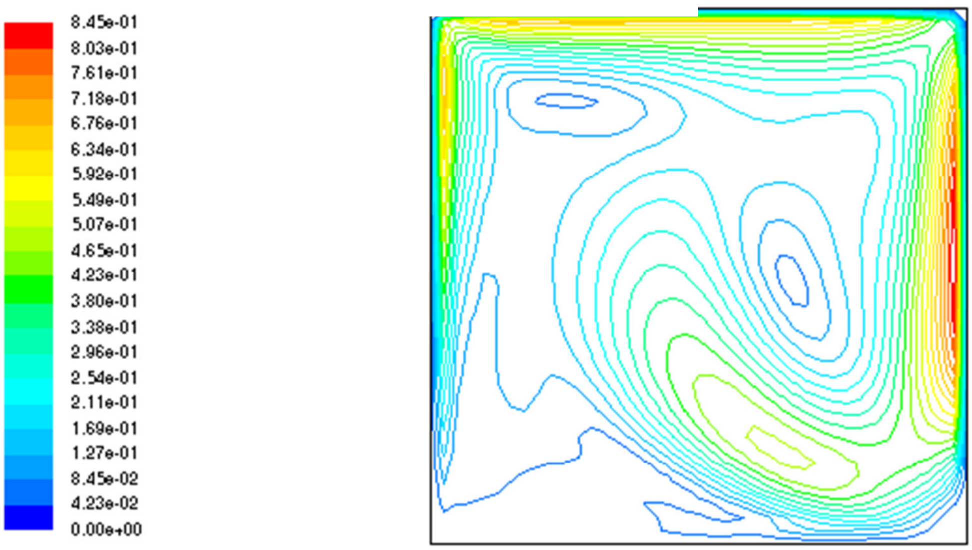


Fig 7.3 Ar = 2 velocity magnitude

7.2 Streamlines

For Ar of 0.5, the vortices are concentrated on the upper region of the hot surface and they move towards the cold surface they break into two. As the aspect ratio increases, the number of vortices on the upper region of the hot surface and bottom region of the cold surface is increased

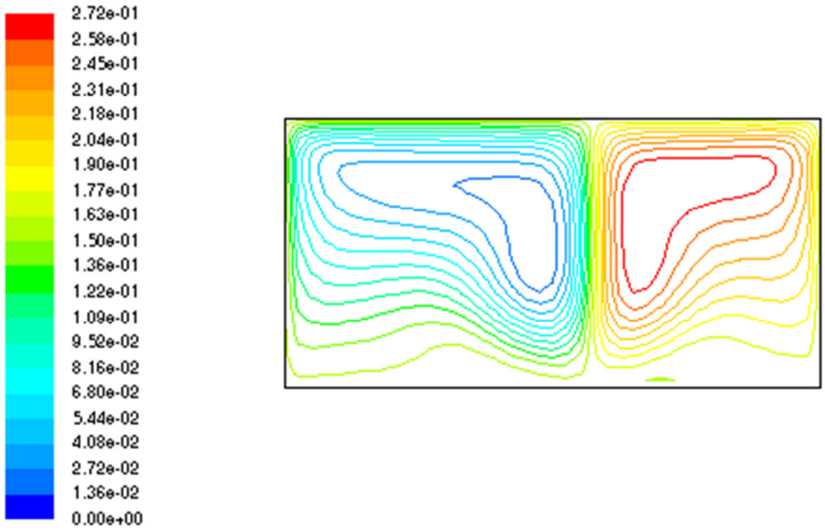


Fig 7.4, Ar = 0.5 streamlines

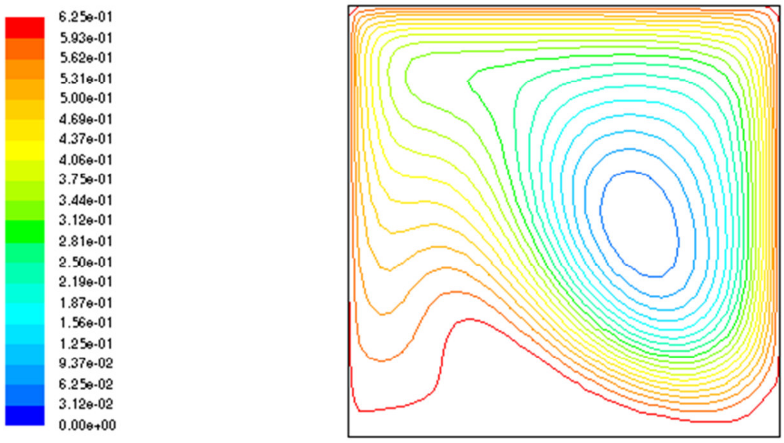


Fig 7.5 Ar= 1 streamlines

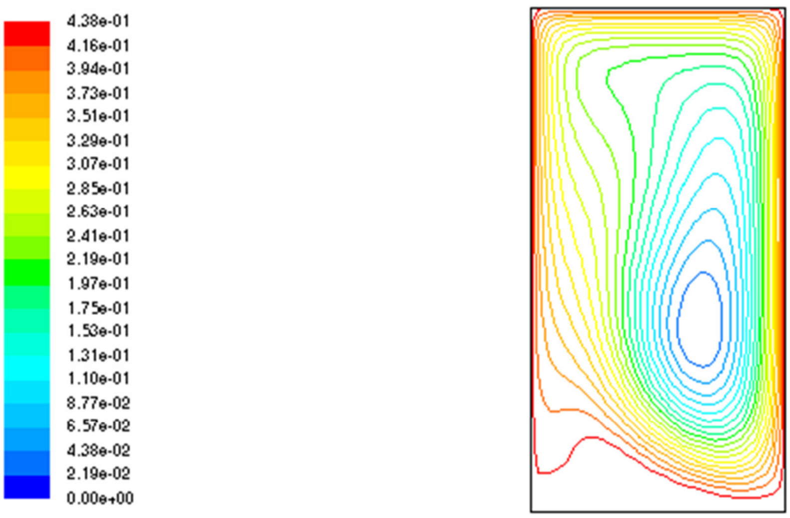


Fig 7.6 Ar= 2 streamlines

7.3 Isotherms

This are lines connecting points of the same temperature. In this study, they have been used to show the temperature distribution in the enclosure. Heat flows from the hot region to the cold region forming layers as the aspect ratio increases.

At low Ar = 0.5, the isotherms are more and evenly distributed in the whole region of the core as seen below. However, on increase in the aspect ratio, the isotherms are concentrated on the upper region of the hot surface and the bottom part of the cold surface.The highest value reached in the isotherms contours is 253K for an aspect ratio of 2 and the lowest is 137K when the aspect ratio is 0.5. The temperature is not evenly distributed in the enclosure as it concentrates on the upper region of the hot surface and the bottom region of the cold surface.

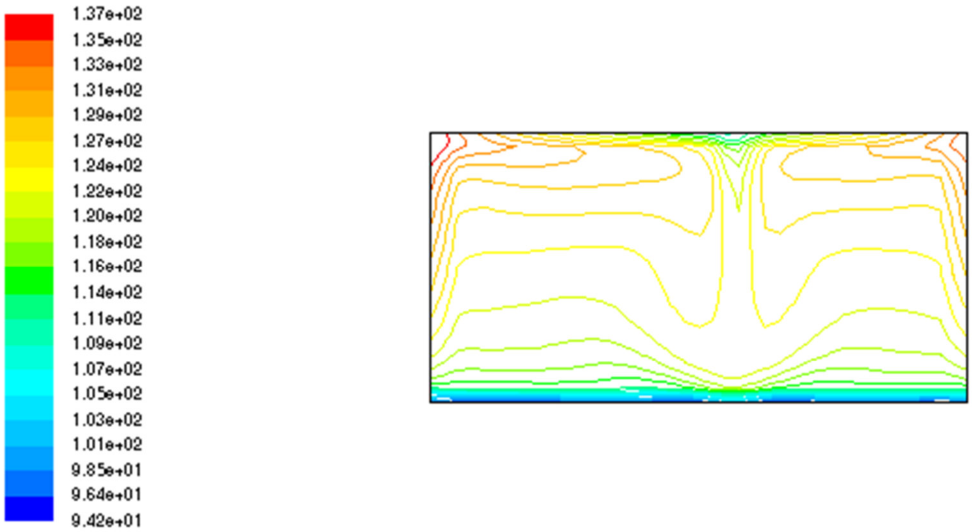


Fig 7.7 Ar= 0.5 Isotherms

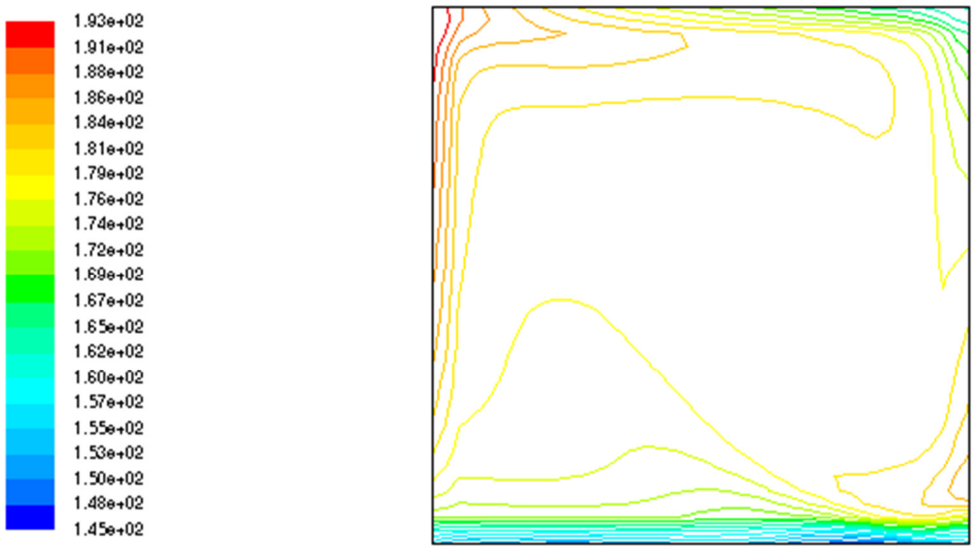


Fig. 7.8 Ar = 1 isotherms

Journal of Natural Sciences Research
ISSN 2224-3186 (Paper) ISSN 2225-0921 (Online)
Vol.8, No.6, 2018

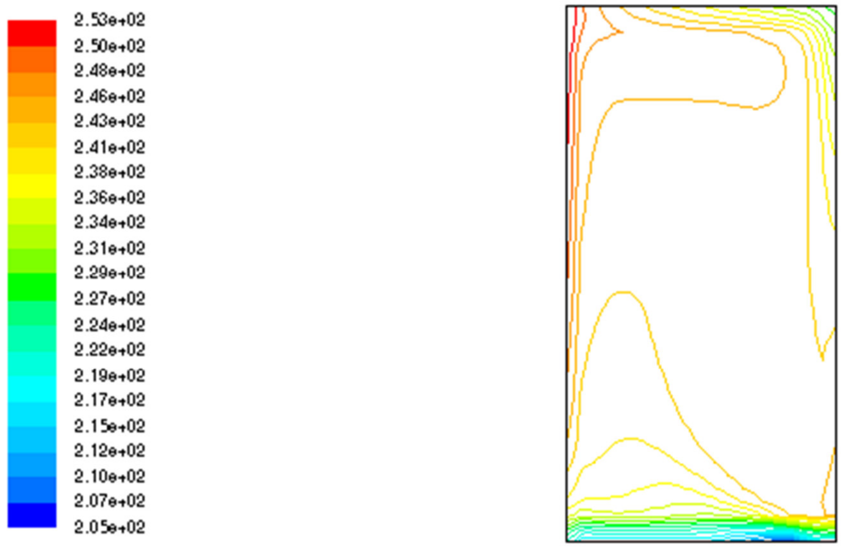


Fig 7.9 Ar = 2 Isotherms

8 CONCLUSION

The rationale of this study was to look effects on heat transfer by convection when varying the aspect ratio of an enclosure. The geometry in consideration is a two dimensional rectangular enclosure filled with air. The temperature difference between the hot surface and cold surface of the enclosure is kept constant at 40K and the aspect ratio varied by varying the height of the enclosure. Heat transfer by convection therefore depends on the length and the height of the enclosure.

9. Nomenclature

Ar	Aspect Ratio
c_p	Specific heat capacity, J/kgK
e	Specific internal energy, J/kg
g	Gravitational acceleration, m/s^2
H	Height of the enclosure, m
k	Kinetic energy of turbulence, m^2/s^2
K	Thermal Conductivity of the fluid, W/m^2
L	Length of the enclosure, m
M	Mass flow, kg/s
P	Thermodynamic Pressure, pa
P_r	Prandtl number
Q	Heat flux, W/m^2
R_a	Rayleigh number
T	Thermodynamic temperature, K
t	Time, s
u, u′, U	Instantaneous velocity components in x – direction, fluctuation velocity and mean velocity in x – direction respectively, m/s
U_*	Characteristic velocity, m/s
v, v', V	Instantaneous velocity component in y – direction, fluctuation velocity and mean velocity in y – direction respectively, m/s
x, y	Horizontal and vertical Cartesian co – ordinates

10. Acknowledgment

My sincere appreciation goes to Dr. Kennedy Awuor (KENYATTA UNIVERSITY) for his guidance and timely encouragement.

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